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ON SOFT KU-ALGEBRAS

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Abstract

In this article, we investigated the algebraic structures of KU-algebras by applying soft set theory. The notions of soft KU-algebras and soft KU-subalgebras is developed, and some basic properties are discussed.

1. Introduction

Molodtsov [8] gave the concept of soft set theory first time, which is very helpful tool for dealing vague and the problems containing uncertainty. Molodtsov introduced several directions for the implementations of soft sets. Now a days soft set theory progressing rapidly. Maji et al. [7], use the soft sets in decision making problems. Soft set theory in the algebraic structure were first implemented by Aktas and Cagman [1]; soft set theory to BCK/BCI-algebras was first applied by Jun and Park [4, 5].

KU-algebra, which is a new algebraic structure introduced by Prabpayak and Leerawat [13]. They gave various related properties in [14]. Mostafa et al. [9] studied KU-algebra in fuzzy context and studied

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fuzzy KU-ideals of KU-algebras and then they investigated several basic properties, which are related to fuzzy KU-ideals, also see [10]. In [11], authors established some interesting results on cubic KU-algebras, and in [12], the results on cubic Γ-hyperideals in left almost Γ-semihypergroups have been developed.

In this article, we apply the concept of soft set theory in KU-algebra and further theory of soft KU-algebras and soft KU-subalgebras are developed, and then their basic properties are investigated.

2. Review of Literature

Now we will recall some known concepts related to KU-algebra and soft sets from the literature, which will be helpful in further study of this article.

Definition 1 ([13]). A KU-algebra is an algebra $(\mathbf{X}, *, 0)$ of type (2, 0) with a single binary operation $*$ that satisfies the following identities:

(1) $(x_1 * x_2) * [(x_2 * x_3) * (x_1 * x_3)] = 0,$

(2) $x * 0 = 0,$

(3) $0 * x = x,$

(4) $x_1 * x_2 = 0 = x_2 * x_1$ implies $x_1 = x_2,$

for any $x_1, x_2, x_3 \in \mathbf{X}$.

In further, we denote a KU-algebra by $\mathbf{X}$. In $\mathbf{X}$, we can define a binary relation $\leq$ by $x_1 \leq x_2$ if and only if $x_2 * x_1 = 0$.

Definition 2 ([14]). A subset S of KU-algebra $\mathbf{X}$ is called "KU-subalgebra" of $\mathbf{X}$ if $x_1 * x_2 \in S,$ whenever $x_1, x_2 \in S$.

Definition 3 ([14]). Let $(\mathbf{X}, *, 0)$ and $(\mathbf{X}', *', 0')$ be KU-algebras. A homomorphism is a map $f : \mathbf{X} \to \mathbf{X}'$ satisfying $f(x_1 * x_2) = f(x_1) *' f(x_2),$ for all $x_1, x_2 \in \mathbf{X}$.

Now we will recall the some basic definitions of soft sets.

Molodtsov defined the notion of a soft sets as follows. Let U be an initial universe and E be the set of parameters. The parameters are usually "attributes, characteristics or properties of an object". Let $P(U)$ denote the power set of U and C is a subset of E.

Definition 4 ([8])**.** A pair $(\mathcal{F}, C)$ is called a soft set over U, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : C \to P(U)$.

In other words, a soft set over U is a parametrized family of subsets of U.

Definition 5 ([6])**.** Let $(\mathcal{F}, C)$ and (G, D) be two soft sets over a common universe U. The soft set $(\mathcal{F}, C)$ is called a soft subset of (G, D), if $C \subseteq D$ and for all $\varepsilon \in C$, $\mathcal{F}[\varepsilon] \subseteq G[\varepsilon]$. This relationship is denoted by $(\mathcal{F}, C) \sqsubseteq (G, D)$. Similarly $(\mathcal{F}, C)$ is called a soft superset of (G, D), if (G, D) is soft subset of $(\mathcal{F}, C)$. This relationship is denoted by $(\mathcal{F}, C) \sqsupseteq (G, D)$. Two soft sets $(\mathcal{F}, C)$ and (G, D) over U are said to be equal, if $(\mathcal{F}, C)$ is a soft subset of (G, D) and (G, D) is a soft subset of $(\mathcal{F}, C)$.

Definition 6 ([6])**.** Let $(\mathcal{F}, C)$ and (G, D) be any two soft sets over U.

(1) The intersection (H, E) of two soft sets $(\mathcal{F}, C)$ and (G, D) is defined as the soft set $(H, E) = (\mathcal{F}, C) \tilde{\bigcap} (G, D)$, where $E = C \bigcap D$ and for all $\varepsilon \in C$

$$H[c] = \begin{cases} \mathcal{F}[\varepsilon], & \text{if } \varepsilon \in C \setminus D, \\ G[\varepsilon], & \text{if } \varepsilon \in D \setminus C, \\ \mathcal{F}[\varepsilon] \bigcap G[\varepsilon], & \text{if } \varepsilon \in C \bigcap D. \end{cases}$$

(2) The union (H, E) of two soft sets (F, C) and (G, D) is defined as the soft set $(H, E) = (F, C) \tilde{\cup} (G, D)$, where $E = C \cup D$ and for all $\varepsilon \in C$

$$H[c] = \begin{cases} F[\varepsilon], & \text{if } \varepsilon \in C \setminus D, \\ G[\varepsilon], & \text{if } \varepsilon \in D \setminus C, \\ F[\varepsilon] \cup G[\varepsilon], & \text{if } \varepsilon \in C \cap D. \end{cases}$$

(3) The AND operation (F, C) AND (G, D) of two soft sets (F, C) and (G, D) is defined as the soft set $(H, E) = (F, C) \tilde{\wedge} (G, D)$, where $H[\alpha, \beta] = F[\alpha] \cap G[\beta]$ for all $(\alpha, \beta) \in C \times D$.

(4) The OR operation (F, C) OR (G, D) of two soft sets (F, C) and (G, D) is defined as the soft set $(H, E) = (F, C) \tilde{\vee} (G, D)$, where $H[\alpha, \beta] = F[\alpha] \cup G[\beta]$ for all $(\alpha, \beta) \in C \times D$.

Definition 7 ([3, 6])**.** Let (F, C) and (G, D) be two soft sets over U. Then,

(1) The $\wedge$-intersection of two soft sets (F, C) and (G, D) is defined as the soft set $(H, E) = (F, C) \wedge (G, D)$ over U, where $E = C \times D$, where $H[\alpha, \beta] = F[\alpha] \cap G[\beta]$ for all $(\alpha, \beta) \in C \times D$.

(2) The $\vee$-union of two soft sets (F, C) and (G, D) is defined as the soft set $(H, E) = (F, C) \vee (G, D)$ over U, where $E = C \times D$, where $H[\alpha, \beta] = F[\alpha] \cup G[\beta]$ for all $(\alpha, \beta) \in C \times D$.

(3) Let (F, C) and (G, D) be two soft sets over G and K, respectively. The Cartesian product of the soft sets (F, C) and (G, D), denoted by $(F, C) \times (G, D)$, is defined as $(F, C) \times (G, D) = (U, A \times B)$, where $U[\alpha, \beta] = F[\alpha] \times G[\beta]$ for all $(\alpha, \beta) \in C \times D$.

3. Soft KU-algebras

Now we introduce the notion of soft KU-algebras. In this article, **X** will denote KU-algebra unless stated otherwise.

Definition 8. A soft set $(\mathcal{F}, C)$ over **X** is called a soft KU-algebra over **X**, if $\mathcal{F}[\varepsilon]$ is a KU-subalgebra of **X**, for all $\varepsilon \in C$.

Proposition 1. *A soft set* $(\mathcal{F}, C)$ *over* **X** *is a soft KU-algebra, if and only if each* $\emptyset \neq \mathcal{F}[\varepsilon]$ *is a KU-subalgebra of* ***X***, *for all* $\varepsilon \in C$.

Proof. Let $(\mathcal{F}, C)$ be a soft KU-algebra over **X**. Then by above definition, $\mathcal{F}[\varepsilon]$ is a KU-subalgebra of **X**, for all $\varepsilon \in C$. It follows that for all $\varepsilon \in C$, $\mathcal{F}[\varepsilon] \neq \emptyset$ is a KU-subalgebra of **X**.

Conversely, let us consider that $(\mathcal{F}, C)$ is a soft set over **X** such that for all $\varepsilon \in C$, $\mathcal{F}[\varepsilon] \neq \emptyset$ is a KU-subalgebra of **X**, whenever $\mathcal{F}[\varepsilon] \neq \emptyset$. Since $\mathcal{F}[\varepsilon]$ is a KU-subalgebra of **X**. Hence $(\mathcal{F}, C)$ is a soft KU-algebra over **X**.

□

Example 1. Let $\mathbf{X} = \{0, a, b, c, d\}$ in which $*$ is defined by the following table:

$*$	0	a	b	c	d
0	0	a	b	c	d
a	0	0	0	0	a
b	0	c	0	c	d
c	0	a	b	0	a
d	0	0	0	0	0

Clearly $(\mathbf{X}, *, 0)$ is a KU-algebra. Let the soft set $(\mathcal{F}, \mathbf{X})$, with $\mathcal{F} : \mathbf{X} \to P(\mathbf{X})$ is defined as $\mathcal{F}(0) = \{0\}$, $\mathcal{F}(a) = \{0, a\}$, $\mathcal{F}(b) = \{0, a, c\}$, $\mathcal{F}(c) = \{0, c, d\}$, and $\mathcal{F}(d) = \{0, d\}$. It is clear that $(\mathcal{F}, \mathbf{X})$ is a soft KU-algebra over **X**.

Example 2. Let $\mathbf{X} = \{0, a, b, c\}$ in which $*$ is defined by

$*$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	0	0	0

Clearly $(\mathbf{X}, *, 0)$ is a KU-algebra. Let us consider the soft set $(\mathcal{F}, \mathbf{X})$, where $\mathcal{F} : \mathbf{X} \to P(\mathbf{X})$ defined by $\mathcal{F}(0) = \{0\}, \mathcal{F}(a) = \{0, a\}, \mathcal{F}(b) = \{0, a, c\}$, $\mathcal{F}(c) = \{a, b, c\}$, then it is clear that $(\mathcal{F}, \mathbf{X})$ is a not soft KU-algebra over $\mathbf{X}$ as $\mathcal{F}(c) = \{a, b, c\}$ is not a KU-subalgebra of $\mathbf{X}$. It shows that there may exist some soft sets, which are not soft KU-algebra.

Theorem 1. *Let $(\mathcal{F}, C)$ and (G, D) be two soft KU-algebras over* $\mathbf{X}$. *Then so is their intersection, i.e.,* $(\mathcal{F}, C) \widetilde{\cap} (G, D)$.

Proof. As $(\mathcal{F}, C)$ and (G, D) are two soft KU-algebras over $\mathbf{X}$, so their intersection over $\mathbf{X}$ is a soft set (H, E), where $E = C \cup D$ and for all $\varepsilon \in E$, it is defined as

$$H[\varepsilon] = \begin{cases} \mathcal{F}[\varepsilon], & \text{if } \varepsilon \in C \setminus D, \\ G[\varepsilon], & \text{if } \varepsilon \in D \setminus C, \\ \mathcal{F}[\varepsilon] \cap G[\varepsilon], & \text{if } \varepsilon \in C \cap D. \end{cases}$$

Since for all $\varepsilon \in E$ either $\varepsilon \in C \setminus D$ or $\varepsilon \in D \setminus C$ or $\varepsilon \in C \cap D$. If $\varepsilon \in C \setminus D$, then $H[\varepsilon] = \mathcal{F}[\varepsilon]$. As $\mathcal{F}[\varepsilon]$ is a KU-algebra over $\mathbf{X}$, then $H[\varepsilon]$ is a KU-algebra over $\mathbf{X}$. If $\varepsilon \in D \setminus C$, then $H[\varepsilon] = G[\varepsilon]$. As $G[\varepsilon]$ is a KU-algebra over $\mathbf{X}$, then $H[\varepsilon]$ is a KU-algebra over $\mathbf{X}$. If $\varepsilon \in C \cap D$, then $H[\varepsilon] = \mathcal{F}[\varepsilon] \cap G[\varepsilon]$. As $\mathcal{F}[\varepsilon]$ and $G[\varepsilon]$ are both KU-algebra over $\mathbf{X}$, then $H[\varepsilon]$ is a KU-algebra over $\mathbf{X}$. In all the three cases, $H[\varepsilon]$ is a KU-algebra over $\mathbf{X}$. Hence $(H, E) = (\mathcal{F}, C) \widetilde{\cap} (G, D)$ is a soft KU-algebra over $\mathbf{X}$. $\square$

Theorem 2. *Let* (F, C) *and* (G, D) *be two soft KU-algebras over* **X**. *Then so is* $(F, C) \widetilde{\cup} (G, D)$.

Proof. The proof is straightforward. □

Theorem 3. *Let* (F, C) *and* (G, D) *be two soft KU-algebras over* **X.** *Then so is* $(F, C) \widetilde{\wedge} (G, D)$.

Proof. Using Definition 6(3), we have

$$(H, C \times D) = (F, C) \widetilde{\wedge} (G, D),$$

where $H[\alpha, \beta] = F[\alpha] \cap G[\beta]$ for all $(\alpha, \beta) \in C \times D$. Since (F, α) and $G[\beta]$ are KU-subalgebras of **X**, the intersection $F[\alpha] \cap G[\beta]$ is also a KU-subalgebra of **X**. Therefore, $H[\alpha, \beta]$ is a KU-subalgebra of **X** for all $(\alpha, \beta) \in C \times D$. Thus $(H, E) = (F, C) \widetilde{\wedge} (G, D)$ is a soft KU-algebra over **X**.

□

Theorem 4. *Let* (F, C) *and* (G, D) *be two soft KU-algebras over* **X**. *Then so is* $(F, C) \widetilde{\vee} (G, D)$.

Proof. The proof is straightforward. □

Definition 9. Let $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, family of soft KU-algebras over **X**, then the bi(restricted)-intersection over a common universe U is defined as the soft set $\widetilde{\sqcap}_{i \in \wedge}(F_i, C_i) = (H, E)$, where $E = \cap_{i \in \wedge} C_i$ and $H(x) = \cap_{i \in \wedge} F_i(x)$ for all $x \in E$.

Theorem 5. *Let* $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, *family of soft KU-algebras over* **X**. *Then the bi-intersection* $\widetilde{\sqcap}_{i \in \wedge}(F_i, C_i)$ *is a soft KU-algebra over* **X**.

Proof. Let $\{(F_i, C_i) | i \in \wedge\}$ be a non-empty family of soft KU-algebras over **X**. By definition of bi-intersection $\widetilde{\sqcap}_{i \in \wedge}(F_i, C_i) = (H, E)$, where $E = \cap_{i \in \wedge} C_i$ and $H(x) = \cap_{i \in \wedge} F_i(x)$, for all $x \in E$. Let

$x \in (H, E)$, then $\bigcap_{i \in \wedge} F_i(x) \neq \emptyset$, so we get $F_i(x) \neq \emptyset$ for all $i \in \wedge$. Since each (F_i, C_i) is a soft KU-algebra over **X**, it follows $F_i(x)$ is a KU-subalgebra of **X** for all $i \in \wedge$, and hence $H(x) = \bigcap_{i \in \wedge} F_i(x)$ is a KU-subalgebra of **X**. So $\widetilde{\sqcap}_{i \in \wedge}(F_i, C_i) = (H, E)$ is a soft KU-algebra over **X**. □

Corollary 1. *Let* $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, *family of soft KU-algebras over* **X**. *Then the bi-intersection* $\widetilde{\sqcap}_{i \in \wedge}(F_i, C_i)$ *is a soft KU-algebra over* **X**.

Proof. The proof is straightforward. □

Definition 10. Let $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, family of soft KU-algebras over **X**. Then the extended-intersection is the soft set defined by $\widetilde{\bigcap}_{i \in \wedge}(F_i, C_i) = (H, E)$, where $E = \bigcup_{i \in \wedge} C_i$ and $H(x) = \bigcap_{i \in \wedge} F_i(x)$ and $\wedge(x) = \{i | i \in C_i\}$, for all $x \in E$.

Theorem 6. *Let* $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, *family of soft KU-algebras over* **X**. *Then the extended-intersection* $\widetilde{\bigcap}_{i \in \wedge}(F_i, C_i)$ *is a soft KU-algebra over* **X**, *if it is non-empty*.

Proof. Let $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, family of soft KU-algebras over **X**. By definition of extended-intersection $\widetilde{\bigcap}_{i \in \wedge}(F_i, C_i) = (H, E)$, where $E = \bigcup_{i \in \wedge} C_i$ and $H(x) = \bigcap_{i \in \wedge} F_i(x)$, for all $x \in E$. Let $x \in (H, E)$, then $\bigcap_{i \in \wedge} F_i(x) \neq \emptyset \Rightarrow F_i(x) \neq \emptyset$ for all $i \in \wedge$. Since each (F_i, C_i) is a soft KU-algebra over **X**, it follows $F_i(x)$ is a KU-subalgebra of **X** for all $i \in \wedge$, and thus $H(x) = \bigcap_{i \in \wedge} F_i(x)$ is a KU-subalgebra of **X**. Hence $\widetilde{\bigcap}_{i \in \wedge}(F_i, C_i) = (H, E)$ is a soft KU-algebras over **X**. □

Definition 11. Let $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, family of soft KU-algebras over **X**. Then the restricted-union is the soft set defined by $\widetilde{\bigcup}_{i \in \wedge}(F_i, C_i)$ $= (H, E)$, where $E = \bigcap_{i \in \wedge} C_i \neq \emptyset$ and $H(x) = \bigcup_{i \in \wedge} F_i(x)$ for all $x \in E$.

Theorem 7. *Let* $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, *family of soft KU-algebras over* **X**. *If* $F_i(x_i) \subseteq F_j(x_j)$ *or* $F_j(x_j) \subseteq F_i(x_i)$ *for all* $i, j \in \wedge$ *and* $x_i \in C_i$, *then the restricted union* $\widetilde{\bigcup}_{i\in\wedge} (F_i, C_i)$ *is a soft KU-algebras over* **X**.

Proof. Let $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, family of soft KU-algebras over **X**. By definition of restricted-union $\widetilde{\bigcup}_{i\in\wedge} (F_i, C_i) = (H, E)$, where $E = \bigcap_{i\in\wedge} C_i$ and $H(x) = \bigcup_{i\in\wedge} F_i(x)$ for all $x \in E$. Let $x \in (H, E)$. Since $\bigcup_{i\in\wedge} F_i(x) \neq \emptyset$, so we get $F_{i_o}(x) \neq \emptyset$ for some $i_o \in \wedge$. Since each (F_i, C_i) is a soft KU-algebra over **X**, it follows $F_{i_o}(x)$ is a KU-subalgebra of **X** for all $i_o \in \wedge$, and thus $H(x) = \bigcup_{i\in\wedge} F_i(x)$ is a KU-subalgebra of **X**. Hence $\widetilde{\bigcup}_{i\in\wedge} (F_i, C_i) = (H, E)$ is a soft KU-algebras over **X**. □

Theorem 8. *Let* $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, *family of soft KU-algebras over* **X**. *Then the* $\wedge$*-intersection* $\widetilde{\wedge}_{i\in\wedge}(F_i, C_i)$ *is a soft KU-algebras over* **X**.

Proof. Let $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, family of soft KU-algebras over **X**. By definition $\widetilde{\wedge}_{i\in\wedge}(F_i, C_i) = (H, E)$, where $E = \prod_{i\in\wedge} C_i$ and $H(x) = \bigcap_{i\in\wedge} F_i(x_i)$ for all $x = (x_i)_{i\in\wedge} \in E$.

Let $x = (x_i)_{i\in\wedge} \in (H, E)$, then $H(x) = \bigcap_{i\in\wedge} F_i(x_i) \neq \emptyset$, so we get $F_i(x) \neq \emptyset$ for all $i \in \wedge$. Since each (F_i, C_i) is a soft KU-algebra over **X**, it follows $F_i(x_i)$ is a KU-subalgebra of **X** for all $i \in \wedge$, and hence $H(x) = \bigcap_{i\in\wedge} F_i(x)$ is a KU-subalgebra of **X**. That is $\widetilde{\wedge}_{i\in\wedge}(F_i, C_i) = (H, E)$ is a soft KU-algebras over **X**. □

Theorem 9. *Let* $\{(F_i, C_i) | i \in \wedge\} \neq \Phi$, *family of soft KU-algebras over* **X**. *If* $F_i(x_i) \subseteq F_j(x_j)$ *or* $F_j(x_j) \subseteq F_i(x_i)$ *for all* $i, j \in \wedge$ *and* $x_i \in C_i$. *Then so is* $\vee-$*union* $\widetilde{\vee}_{i\in\wedge}(F_i, C_i)$ *is a soft KU-algebras over* **X**.

Proof. The proof is straightforward. □

Definition 12. Let $\{(\mathcal{F}_i, C_i) \mid i \in \wedge\} \neq \Phi$, family of soft KU-algebras over **X**. Then we define $\widetilde{\prod}_{i\in\wedge}(\mathcal{F}_i, C_i) = (H, E)$, where $E = \prod_{i\in\wedge} C_i$ and $H(x) = \prod_{i\in\wedge} \mathcal{F}_i(x_i)$, for all $x = (x_i)_{i\in\wedge} \in E$, which is known as Cartesian product of soft sets.

Theorem 10. *Let* $\{(\mathcal{F}_i, C_i) \mid i \in \wedge\} \neq \Phi$, *family of soft KU-algebras over* **X**. *Then so is the Cartesian product* $\widetilde{\prod}_{i\in\wedge}(\mathcal{F}_i, C_i)$.

Proof. By definition, we can write $\widetilde{\prod}_{i\in\wedge}(\mathcal{F}_i, C_i) = (H, E)$, where $E = \prod_{i\in\wedge} C_i$ and $H(x) = \prod_{i\in\wedge} \mathcal{F}_i(x_i)$ for all $x = (x_i)_{i\in\wedge} \in E$. Let $x = (x_i)_{i\in\wedge} \in (H, E)$, then $H(x) = \widetilde{\prod}_{i\in\wedge}(\mathcal{F}_i, C_i) \neq \emptyset$, so we get $\mathcal{F}_i(x_i) \neq \emptyset$ for all $i \in \wedge$. Since each $(\mathcal{F}_i, C_i)$ is a soft KU-algebra over **X**, it follows $\mathcal{F}_i(x_i)$ is a KU-subalgebra of **X** for all $i \in \wedge$, and hence $H(x) = \widetilde{\prod}_{i\in\wedge} \mathcal{F}_i(x)$ is a KU-subalgebra of **X**. Hence, the Cartesian product $\widetilde{\prod}_{i\in\wedge}(\mathcal{F}_i, C_i)$ is a soft KU-algebras over $\prod_{i\in\wedge} \mathbf{X}_i$. □

Definition 13. Let $(\mathcal{F}, C)$ be a soft KU-algebra over **X**. Then $(\mathcal{F}, C)$ is called "trivial soft KU-algebra" over **X** if "$\mathcal{F}(x) = \{0\}$" and $(\mathcal{F}, C)$ is called "whole soft KU-algebra" over **X** if "$\mathcal{F}(x) = \mathbf{X}$", for all $x \in C$.

Example 3. Let $\mathbf{X} = \{0, a, b, c\}$ in which $*$ is defined by

$*$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	0	0	0

then $(\mathbf{X}, *, 0)$ is a KU-algebra. Consider the soft set $(\mathcal{F}, C)$, where $C = \{a, b, c\}$. Then $\mathcal{F}(x) = \mathbf{X}$ for all $x \in C$. It is clear that $(\mathcal{F}, C)$ is a "whole soft KU-algebra" over **X**.

Now we consider the soft set $(\mathcal{F}, D)$, where $D = \{b\}$, then $\mathcal{F}(x) = \{0\}$ for all $x \in D$, then it is clear that $(\mathcal{F}, D)$ is a “trivial soft KU-algebra” over **X**.

Definition 14. Let “**X**, **Y** be two KU-algebras” and $\lambda : \mathbf{X} \to \mathbf{Y}$ be a mapping of KU-algebras. If $(\mathcal{F}, C)$ and (G, D) are any two soft sets over **X** and **Y**, respectively, then the image of the soft set $(\mathcal{F}, C) = (\lambda(\mathcal{F}), C)$ is a soft set over **Y**, where $\lambda(\mathcal{F}) : C \to P(y)$ is defined as $\lambda(\mathcal{F})(x) = \lambda(\mathcal{F}(x))$ for all $x \in C$ and the inverse image of the soft set $(G, D) = (\lambda^{-1}(G), D)$ is a soft set over **X**, where $\lambda^{-1}(G) : D \to P(\mathbf{X})$ is defined by $\lambda^{-1}(G)(y) = \lambda^{-1}(G(y))$ for all $y \in D$.

Lemma 1. *Let* $\lambda : \mathbf{X} \to \mathbf{Y}$ *be an onto homomorphism of KU-algebras*,

(i) *if* $(\mathcal{F}, C)$ *is a soft KU-algebra over* **X**, *then* $(\lambda(\mathcal{F}), C)$ *is a soft KU-algebra over* **Y**;

(ii) *if* (G, D) *is a soft KU-algebra over* **Y**, *then* $(\lambda^{-1}(G), D)$ *is a soft KU-algebra over* **X**.

Proof. (i) Since $(\mathcal{F}, C)$ is a soft KU-algebra over **X**, so $(\lambda(\mathcal{F}), C)$ is a non-empty soft set over **Y**. Let $x \in (\lambda(\mathcal{F}), C)$, we have $\lambda(\mathcal{F})(x) = \lambda(\mathcal{F}(x)) \neq \emptyset$. As $\mathcal{F}(x)$ is a KU-subalgebra over **X**, its onto homomorphic image $\lambda(\mathcal{F}(x))$ is a KU-subalgebra over **Y**. Hence $\lambda(\mathcal{F}(x))$ is a KU-subalgebra over **Y** for all $x \in C$. Thus $(\lambda(\mathcal{F}), C)$ is a soft KU-algebra over **Y**.

(ii) Since $(\lambda^{-1}(G), D) \subset (G, D)$. Let $y \in (\lambda^{-1}(G), D)$, then $G(y) \neq \emptyset$. As $G(y) \neq \emptyset$ is a subalgebra over **Y**, its onto homomorphic inverse image $\lambda^{-1}(G)(y)$ is a KU-algebra over **X**. Hence $\lambda^{-1}(G)(y)$ is a KU-subalgebra over **X** for all $y \in D$. Thus $(\lambda^{-1}(G), D)$ is a soft KU-algebra over **X**. □

Theorem 11. *Let* $\lambda : \mathbf{X} \to \mathbf{Y}$ *be an onto homomorphism mapping of KU-algebras. Let* $(\mathcal{F}, C)$ *and* (G, D) *are two soft KU-algebras over* $\mathbf{X}$ *and* $\mathbf{Y}$, *respectively, then*

(i) *if* $\mathcal{F}(x) = \ker(\lambda)$ *for all* $x \in C$, *then* $(\lambda(\mathcal{F}), C)$ *is a trivial soft KU-algebra over* $\mathbf{Y}$;

(ii) *if* λ *is onto and* $(\mathcal{F}, C)$ *is whole soft KU-algebra over* $\mathbf{X}$, *then* $(\lambda(\mathcal{F}), C)$ *is a whole soft KU-algebra over* $\mathbf{Y}$;

(iii) *if* $G(y) = \lambda(X)$ *for all* $y \in D$, *then* $(\lambda^{-1}(G), D)$ *is a whole soft KU-algebra over* $\mathbf{X}$;

(iv) *if* λ *is injective and* (G, D) *is trivial soft KU-algebra over* $\mathbf{Y}$, *then* $(\lambda^{-1}(G), D)$ *is a trivial soft KU-algebra over* $\mathbf{X}$.

Proof. (i) Let $\mathcal{F}(x) = \ker(\lambda)$ for all $x \in C$, then $\lambda(\mathcal{F})(x) = \lambda(\mathcal{F}(x)) = (0_Y)$ for all $x \in C$. Hence by above lemma, $(\lambda(\mathcal{F}), C)$ is a trivial soft KU-algebra over $\mathbf{Y}$.

(ii) Suppose λ is onto and $(\mathcal{F}, C)$ is whole soft KU-algebra over $\mathbf{X}$. Then $\mathcal{F}(x) = \mathbf{X}$ for all $\mathbf{X} \in C$, and so $\lambda(\mathcal{F})(\mathbf{X}) = \lambda(\mathcal{F}(x)) = \lambda(X) = \mathbf{Y}$, for all $x \in C$. By above lemma, $(\lambda(\mathcal{F}), C)$ is a whole soft KU-algebra over $\mathbf{Y}$.

(iii) Let $G(x) = \lambda(x)$, for all $y \in D$. Then $\lambda^{-1}(G)(y) = \lambda^{-1}(G(y)) = \lambda^{-1}(\lambda(\mathbf{X})) = \mathbf{X}$, for all $y \in D$. So by above lemma, $(\lambda^{-1}(G), D)$ is a whole soft KU-algebra over $\mathbf{X}$.

(iv) Let $(\lambda^{-1}(G), D)$ is a trivial soft KU-algebra over $\mathbf{X}$, then $G(y) = \{O\}$ for all $(\lambda^{-1}(G), D)$ is a whole soft KU-algebra over $\mathbf{X}$. So $\lambda^{-1}(G)(y) = \lambda^{-1}(G(y)) = \lambda^{-1}(\{O\}) = \ker(\lambda) = \{O_X\}$, for all $y \in D$. Thus by above lemma, $(\lambda^{-1}(G), D)$ is a trivial soft KU-algebra over $\mathbf{X}$. $\square$

Definition 15. Let $(\mathcal{F}, C)$ and (G, D) be two soft KU-algebras over $\mathbf{X}$. Then (G, D) is called soft KU-subalgebra of $(\mathcal{F}, C)$, denoted by $(G, D)\ \tilde{\sqsubset}_s (\mathcal{F}, C)$, if it satisfies the following conditions:

(i) $D \subseteq C$,

(ii) $G(x)$ is a KU-subalgebra of $\mathcal{F}(x)$ for all $x \in (G, D)$.

Example 4. Let $\mathbf{X} = \{0, a, b, c\}$ in which $*$ is defined by the following table:

$*$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	0	0	0

Clearly $(\mathbf{X}, *, 0)$ is a KU-algebra. Consider the soft set $(\mathcal{F}, C)$, where $C = \{a, b, c\}$, then $\mathcal{F}(x) = \mathbf{X}$ for all $x \in C$. It is clear that $(\mathcal{F}, \mathbf{X})$ is soft KU-algebra over $\mathbf{X}$.

Also consider the soft set (G, D), where $D = \{b\}$, then $G(x) = \{0\}$ for all $x \in D$. It is clear that (G, D) is a soft KU-algebra over $\mathbf{X}$. By above definition, $D \subseteq C$ and $G(x)$ is a KU-subalgebra of $\mathcal{F}(x)$ for all $x \in (G, D)$. Hence $(G, D) \tilde{\sqsubset}_s (\mathcal{F}, C)$.

Theorem 12. *Let $(\mathcal{F}, C)$ be a soft KU-algebras over $\mathbf{X}$, if $E \subset C$, then $(\mathcal{F}|_E, E)$ is a soft KU-algebra over $\mathbf{X}$.*

Proof. The proof is straightforward. $\square$

Example 5. Let $\mathbf{X} = \{0, a, b, c\}$ in which $*$ is defined by the following table:

$*$	0	a	b	c
0	0	a	b	c
a	0	0	b	c
b	0	a	0	c
c	0	0	0	0

Clearly $(\mathbf{X}, *, 0)$ is a KU-algebra. Consider the soft set $(\mathcal{F}, C)$, where $\mathcal{F} : C \to P(\mathbf{X})$ is defined as $\mathcal{F}(x) = \{a, b, c\}$ for all $x \in C$ is not a subalgebra of $\mathbf{X}$. So $(\mathcal{F}, C)$ is not a soft KU-algebra over $\mathbf{X}$. But, if we take $E \subset C = \{0\}$, then $(\mathcal{F}, E)$ is a soft KU-algebra over $\mathbf{X}$.

Theorem 13. *Let $(\mathcal{F}, C)$ and (G, D) be two soft KU-algebras over* $\mathbf{X}$ *and $(G, D) \tilde{\subseteq} (\mathcal{F}, C)$, then $(G, D) \tilde{\sqsubset}_s (\mathcal{F}, C)$.*

Proof. The proof is straightforward. □

Theorem 14. *Let $(\mathcal{F}, C)$ be a soft KU-algebra over* $\mathbf{X}$ *and $\{(H_i, C_i) | i \in \wedge\}$ be a non-empty family of soft KU-subalgebras of $(\mathcal{F}, C)$. Then the bi-intersection $\tilde{\sqcap}_{i \in \wedge}(H_i, C_i)$ is a soft KU-subalgebras of $(\mathcal{F}, C)$, if it is non-empty.*

Proof. The proof is straightforward. □

Theorem 15. *Let $(\mathcal{F}, C)$ be a soft KU-algebra over* $\mathbf{X}$ *and $\{(H_i, C) | i \in \wedge\}$ be a non-empty family of soft KU-subalgebras of $(\mathcal{F}, C)$. Then the bi-intersection $\tilde{\sqcap}_{i \in \wedge}(H_i, C)$ is a soft KU-subalgebras of $(\mathcal{F}, C)$, if it is non-empty.*

Proof. The proof is straightforward. □

Theorem 16. *Let* $(\mathcal{F}, C)$ *be a soft KU-algebra over* $\mathbf{X}$ *and* $\{(H_i, C_i) | i \in \wedge\}$ *be a non-empty family of soft KU-subalgebras of* $(\mathcal{F}, C)$. *Then the extended-intersection* $\widetilde{\bigcap}_{i\in\wedge} (H_i, C_i)$ *is a soft KU-subalgebras of* $(\mathcal{F}, C)$, *if it is non-empty.*

Proof. The proof is straightforward. □

Theorem 17. *Let* $(\mathcal{F}, C)$ *be a soft KU-algebra over* $\mathbf{X}$ *and* $\{(H_i, C_i) | i \in \wedge\}$ *be a non-empty family of soft KU-subalgebras of* $(\mathcal{F}, C)$. *If* $H_i(x_i) \subseteq H_j(x_j)$ *or* $H_j(x_j) \subseteq H_i(x_i)$ *for all* $i, j \in \wedge$ *and* $x_i \in C_i$, *then the restricted union* $\widetilde{\bigcup}_{i\in\wedge} (H_i, C_i)$ *is a soft KU-algebras of* $(\mathcal{F}, C)$, *if it is non-empty.*

Proof. Let $\{(H_i, C_i) | i \in \wedge\}$ be a non-empty family of soft KU-subalgebras over $\mathbf{X}$. By definition of restricted-union $\widetilde{\bigcup}_{i\in\wedge} (H_i, C_i) = (H, E)$, where $C = \bigcap_{i\in\wedge} C_i$ and $H(x) = \bigcup_{i\in\wedge} \mathcal{F}_i(x)$ for all $x \in C$. Let $x \in (H, E)$. Since $H(x) = \bigcup_{i\in\wedge} \mathcal{F}_i(x) \neq \emptyset$, so we get $H_{i_o}(x) \neq \emptyset$, for some $i_o \in \wedge$. Since $H_i(x_i) \subseteq H_j(x_j)$ or $H_j(x_j) \subseteq H_i(x_i)$, for all $i, j \in \wedge$ and $x_i \in C_i$ and hence $H(x) = \bigcup_{i\in\wedge} \mathcal{F}_i(x)$ is a KU-subalgebra of $\mathbf{X}$ for all $x \in (H, E)$. That is $\widetilde{\bigcup}_{i\in\wedge} (\mathcal{F}_i, C_i) = (H, E)$ is a soft KU-subalgebra of $(\mathcal{F}, C)$. □

Theorem 18. *Let* $(\mathcal{F}, C)$ *be a soft KU-algebra over* $\mathbf{X}$ *and* $\{(H_i, C_i) | i \in \wedge\}$ *be a non-empty family of soft KU-subalgebras of* $(\mathcal{F}, C)$. *Then the* $\wedge$*-intersection* $\widetilde{\wedge}_{i\in\wedge}(H_i, C_i)$ *is a soft KU-subalgebras of* $\widetilde{\wedge}_{i\in\wedge}$ $(\mathcal{F}, C)$.

Proof. The proof is straightforward. □

Theorem 19. *Let* $(\mathcal{F}, C)$ *be a soft KU-algebra over* $\mathbf{X}$ *and* $\{(H_i, C_i) | i \in \wedge\}$ *be a non-empty family of soft KU-subalgebras of* $(\mathcal{F}, C)$. *If* $H_i(x_i) \subseteq H_j(x_j)$ *or* $H_j(x_j) \subseteq H_i(x_i)$ *for all* $i, j \in \wedge$ *and* $x_i \in C_i$, *then the* $\vee$*-union* $\widetilde{\vee}_{i \in \wedge}(H_i, C_i)$ *is a soft KU-subalgebras of* $\widetilde{\vee}_{i \in \wedge}(\mathcal{F}, C)$.

Proof. The proof is straightforward. □

Theorem 20. *Let* $(\mathcal{F}, C)$ *be a soft KU-algebra over* $\mathbf{X}$ *and* $\{(H_i, C_i) | i \in \wedge\}$ *be a non-empty family of soft KU-subalgebras of* $(\mathcal{F}, C)$. *Then the Cartesian product* $\widetilde{\prod}_{i \in \wedge}(H_i, C_i)$ *is a soft KU-subalgebras of* $\widetilde{\prod}_{i \in \wedge}(\mathcal{F}, C)$.

Proof. By definition, we can write $\widetilde{\prod}_{i \in \wedge}(H_i, C_i) = (H, E)$, where $C = \prod_{i \in \wedge} C_i$ and $H(x) = \prod_{i \in \wedge} H_i(x_i) \neq \emptyset$ for all $x = (x_i)_{i \in \wedge} \in E$. Since $\{(H_i, C_i) | i \in \wedge\}$ be a non-empty family of soft KU-subalgebras of $(\mathcal{F}, C)$ we have $H_i(x_i)$ is a KU-subalgebra of $\mathcal{F}(x_i)$ and hence $H(x) = \widetilde{\prod}_{i \in \wedge} H_i(x_i)$ is a KU-subalgebra of $\widetilde{\prod}_{i \in \wedge} \mathcal{F}(x_i)$. That is the Cartesian product $\widetilde{\prod}_{i \in \wedge}(H_i, C_i)$ is a soft KU-subalgebras of $\widetilde{\prod}_{i \in \wedge}(\mathcal{F}, C)$.

□

Theorem 21. *Let* $\lambda : \mathbf{X} \to \mathbf{Y}$ *be a homomorphism of KU-algebras and* $(\mathcal{F}, C)$ *and* (G, D) *be two soft KU-algebras over* $\mathbf{X}$. *If* $(G, D) \widetilde{\sqsubset}_s (\mathcal{F}, C)$, *then*

$$(\mathcal{F}(G), D) \widetilde{\sqsubset}_s (\mathcal{F}(\mathcal{F}), C).$$

Proof. Let $(G, D) \widetilde{\sqsubset}_s (\mathcal{F}, C)$ and $x \in (G, D)$, then $x \in (\mathcal{F}, C)$. By Definition 15, $D \subseteq C$ and $G(x)$ is a KU-subalgebra of $\mathcal{F}(x)$ for all $x \in (G, D)$. Since $\mathcal{F}$ is homomorphism so $\mathcal{F}G(x) = \mathcal{F}(G(x))$ is a KU-subalgebra of $\mathcal{F}(\mathcal{F}(x)) = \mathcal{F}\mathcal{F}(x)$. Hence $(\mathcal{F}(G), D) \widetilde{\sqsubset}_s (\mathcal{F}(\mathcal{F}), C)$. □

Theorem 22. *Let* $\lambda : \mathbf{X} \to \mathbf{Y}$ *be a homomorphism of KU-algebras and* $(\mathcal{F}, C)$ *and* (G, D) *be two soft KU-algebras over* $\mathbf{X}$. *If* $(G, D) \tilde{\sqsubset}_s (\mathcal{F}, C)$, *then*

$$(\mathcal{F}^{-1}(G), D) \tilde{\sqsubset}_s (\mathcal{F}^{-1}(\mathcal{F}), C).$$

Proof. Let $(G, D) \tilde{\sqsubset}_s (\mathcal{F}, C)$ and $y \in (\mathcal{F}^{-1}(G), D)$. By Definition 15, $D \subseteq C$ and $G(y)$ is a KU-subalgebra of $\mathcal{F}(y)$ for all $y \in D$. Since $\mathcal{F}$ is homomorphism so $\mathcal{F}^{-1}G(y) = \mathcal{F}^{-1}(G(y))$ is a KU-subalgebra of $\mathcal{F}^{-1}(\mathcal{F}(y)) = \mathcal{F}^{-1}\mathcal{F}(y)$. Hence $(\mathcal{F}^{-1}(G), D) \tilde{\sqsubset}_s (\mathcal{F}^{-1}(\mathcal{F}), C)$. □

Definition 16. Let $(\mathcal{F}, C)$ and (G, D) be two soft KU-algebras over the KU-algebras $\mathbf{X}$, $\mathbf{Y}$, respectively. Let $\lambda : \mathbf{X} \to \mathbf{Y}$ and $\gamma : C \to D$ be two mappings, then the pair (λ, γ) is called soft function from $(\mathcal{F}, C)$ to (G, D).

A pair (λ, γ) is called soft homomorphism from $\mathbf{X}$ to $\mathbf{Y}$, if it satisfies the following:

(i) λ is homomorphism;

(ii) γ is a mapping;

(iii) $\lambda\mathcal{F}(x) = G\gamma(x)$ for all $x \in C$.

$(\mathcal{F}, C)$ is said to be softly homomorphic to (G, D) under the soft homomorphism (λ, γ), if (λ, γ) is soft homomorphism and both λ, γ are surjective if λ is an isomorphism from $\mathbf{X}$ to $\mathbf{Y}$, and γ is bijective from C to D, then (λ, γ) is called soft isomorphism and that $(\mathcal{F}, C)$ is softly isomorphic to (G, D) under the soft isomorphism (λ, γ). It is denoted by $(\mathcal{F}, C) \sim (G, D)$.

Proposition 2. *The relation "~" is an equivalence relation of soft KU-algebras.*

Proof. The proof is straightforward. □

Example 6. Let $\mathbf{X} = \{0, a, b, c, d\}$ in which $*$ is defined by the following table:

$*$	0	a	b	c	d
0	0	a	b	c	d
a	0	0	0	0	a
b	0	c	0	c	d
c	0	a	b	0	a
d	0	0	0	0	0

Clearly $(\mathbf{X}, *, 0)$ is a KU-algebra. Consider the soft set $(\mathcal{F}, N)$, where N is the set of natural numbers and $\mathcal{F}, N \to P(\mathbf{X})$ is defined as $\mathcal{F}(n) = \begin{Bmatrix} \{0, a, c\}, \text{ if } 2 \mid n \\ \\ \{0, a\}, \quad \text{if } 2 \nmid n \end{Bmatrix}$. Clearly $(\mathcal{F}, N)$ is a soft KU-algebra. Also the soft set (G, N), where N is the set of natural numbers and $G : N \to P(\mathbf{X})$ is defined as $\mathcal{F}(n) = \begin{Bmatrix} \{0, a\}, \text{ if } 2 \mid n \\ \\ \emptyset, \qquad \text{if } 2 \nmid n \end{Bmatrix}$. Clearly (G, N) is a soft KU-algebra. Let $\lambda : \mathbf{X} \to \mathbf{Y}$ defined by $\lambda(x) = \begin{Bmatrix} c, & \text{if } x = c \\ \\ 0, & \text{if } x \neq c. \end{Bmatrix}$.

Clearly λ is a KU-homomorphism. Also, consider the mapping $\gamma : N \to N$ defined by $\gamma(x) = 2x$, then $\lambda(\mathcal{F}(x)) = G(x) = G(\gamma(x))$ for every $x \in N$. Thus (λ, γ) is a soft homomorphism by above definition.

Theorem 23. *Let* $\lambda : \mathbf{X} \to \mathbf{Y}$ *be an onto homomorphism of KU-algebras and* $(\mathcal{F}, C)$ *and* (G, D) *be two soft KU-algebras over* $\mathbf{X}$ *and* $\mathbf{Y}$, *respectively.*

(i) *The soft function* (λ, I_C) *from* $(\mathcal{F}, C)$ *to* (H, C) *is a soft homomorphism from* $\mathbf{X}$ *to* $\mathbf{Y}$, *where* $I_C : C \to C$ *and* $H : C \to P(Y)$ *defined by* $H(x) = \lambda(\mathcal{F}(x))$, *for all* $x \in C$.

(ii) *If* $\lambda : \mathbf{X} \to \mathbf{Y}$ *be an isomorphism, then the soft function* (λ^{-1}, I_D) *from* (G, D) *to* (K, D) *is a soft isomorphism from* $\mathbf{Y}$ *to* $\mathbf{X}$, *where* $I_D : D \to D$ *and* $K : D \to P(\mathbf{X})$ *defined by* $K(x) = \lambda^{-1}(G(x))$, *for all* $x \in D$.

Proof. The proof is straightforward. □

Proposition 3. *Let* $\mathbf{X}$, $\mathbf{Y}$, *and* $\mathbf{Z}$ *be KU-algebras and* $(\mathcal{F}, C)$, (G, D), *and* (H, E) *soft KU-algebras over* $\mathbf{X}$, $\mathbf{Y}$, *and* $\mathbf{Z}$, *respectively. If the soft function* (λ, γ) *from* $(\mathcal{F}, C)$ *to* (G, D) *is a soft homomorphism and the soft function* (η, μ) *is a soft homomorphism from* (G, D) *to* (H, E), *then the soft function* $(\lambda o \eta, \gamma o \mu)$ *is a soft homomorphism from* $(\mathcal{F}, C)$ *to* (H, E).

Proof. The proof is straightforward. □

Theorem 24. *Let* $\mathbf{X}$ *and* $\mathbf{Y}$ *be KU-algebras and* $(\mathcal{F}, C)$, (G, D) *soft sets over* $\mathbf{X}$ *and* $\mathbf{Y}$, *respectively. If* $(\mathcal{F}, C)$ *is a soft KU-algebra over* $\mathbf{X}$ *and* $(\mathcal{F}, C) \sim (G, D)$, *then* (G, D) *is also a soft KU-algebra over* $\mathbf{Y}$.

Proof. The proof is straightforward. □

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